CAPACITATED VEHICLE ROUTING PROBLEM WITH CO₂ RATES FOR URBAN TRANSPORT

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Abstract: Paper discusses problem of routing freight vehicles in urban areas according to criteria of CO₂ emission and traveling time. Time-dependant traffic conditions for urban areas are defined. Ecological aspects of carbon dioxide emission in urban conditions are a base of mathematical model of time-dependent capacitated vehicle routing problem with CO₂ emission level as a criteria function. Model comprises characteristics of urban transport infrastructure, time-dependent traffic density, and different types of vehicles according to the task requirements. Model is implemented onto representation of real transport network and solved for given data with two step-heuristics basing on modified A-star algorithm and genetic programming. Appropriate results, future research directions and other potential uses are presented and discussed.

Keywords: CO₂ emission, Time-Dependent Vehicle Routing Problem, CVRP, city logistics

1. INTRODUCTION

Vehicle routing in highly developed urban areas is multifaceted problem combining technological, economic, environmental, and health issues. Vehicle routing as a purposeful planning for commercial reasons is applied for public transport and for goods distribution to minimize realization cost while productivity is maximized. Fast and reliable freight transport favors urbanization, but it also affects quality of life through emission of harmful exhaust components, noise and congestion. These factors can’t be ignored. Organizing ecological and efficient freight transport must be then a priority.

Among other harmful components of exhaust gases produced by heavy goods vehicles, the carbon dioxide is one of the most significant and can be used in future research as a point of reference to find emission of other air pollutants. The relation between CO, NOₓ and SO₂, and CO₂ has been well documented in a wide range of case studies.
Vehicle routing in urban area must take into account specific city conditions which mean operating within the network of streets loaded by time-dependent traffic, and of determined characteristics. Many additional restrictions are imposed. Freight transport in cities has limited access to the selected areas. It must keep specified delivery hours (time windows) or require high European Emission Standards for vehicles. The structure of transport network changes, and small local reconfiguration like a closed part of street can cause significant disturbances in traffic density. These factors limit network transition time, influence emission of CO₂ (by influencing velocity and acceleration), and induce a necessity of developing tools supporting decisions within vehicle routing in urban areas.

Lowering CO₂ emission from freight transport can be achieved by decreasing total mileage through rational vehicle routing, what is not only economically justified but also leads to perceiving transport companies as green.

Model presented in the paper concerns direct supplies from central depot to customers scattered in the city. Delivery time is related to traffic conditions and differs for particular hours and areas. While modeling urban conditions above elements will be considered:
1. The city is divided into areas according to traffic volume, flow capacity and accessibility. Areas can be outlined for city center or i.e. historical old town.
2. Characteristics of transport links such as length, number of lines, speed limits, area it belongs to, type of allowed traffic, and other are known.
3. Coefficients decreasing velocity estimated for a different time of day are known.
4. Types of vehicle (according to loading capacity) are defined.

The solution of the problem formulated in the paper is a route minimizing CO₂ emission and delivery time. Time-dependant conditions are considered, so the problem is a version of Time Dependent Vehicle Routing Problem (as proposed in [19]).

2. ON-ROAD VEHICLES CO₂ EMISSION

The most important vehicle ecological characteristic is on-road exhaust emission derived from the emission rate against travelled distance. Greenhouse gasses, and in particular CO₂ are the most concerning as they have direct consequences on human health, and indirect ones [3]. Anthropogenic CO₂ is believed to be the primary cause of global warming. Recent investigations (i.e. studies like [9]) show that CO₂ concentrations in the atmosphere has risen from pre-industrial 280 ppm to present levels of ~380 ppm. This increase is attributed to the world’s expanding use of fossil fuels, such as burning of carbon-based fuels for transportation needs. Reports indicate that the vehicle emissions constitute the major source of atmospheric CO₂ in urban areas (around 10% of the total global and 20% of the European atmospheric CO₂ emissions – Nejadkoorki et al., [21]).

CO₂ is emitted in direct proportion to fuel consumption, with a variation by type of fuel [8]. For most vehicles, fuel consumption decrease as vehicle speed reaches 88 or 105 kph and then begins to increase again. Also congestion has a great impact on emissions. In real driving conditions, there is a rapid nonlinear growth in emissions and fuel consumption as travel speeds fall below 38 kph to 20 kph or when it drops from 20 kph to 8 kph [6].
Different fuels are characterized by different carbon content, so the emission rates are different. The complete combustion of 1 kg of diesel fuel produces about 3,11 kg of CO₂. Current research (i.e. [3], [6], [20], [24]) show that emission pattern results from driving style, so it will depend on speed and acceleration course as well as engine torque and rotation speed.

Proper traffic control management strategy will likely change a vehicle’s modal events in the traffic network and thus influence a vehicle’s instantaneous speed profile. Different speed profiles will potentially result in different vehicle exhaust emission [24].

3. CVRP WITH EMISSION RATES

The issue in title belongs to wide class of optimization problems undertaken in literature. Time-Dependant Vehicle Routing Problem discussed in this paper is a version of a Capacitated VRP based on relaxed TSP, for which constrain restricting visiting nodes only once is omitted. In city conditions, passing the same crossroad more than once is common. Founding shortest paths leads to shortening transport cycles and lowering costs, so it also can be adapted to lower CO₂ emission.

The VRP with emission as a criterion is discussed in the literature. Bektaş and Laporte [3] present the Pollution-Routing Problem expanding classical VRP and discuss economies of environmental-friendly vehicle routing. They state that contrary to the VRP, the PRP is significantly more difficult to solve to optimality. Kuo and Wang [15] present approach to optimize the routing plan with minimizing fuel consumption with a Tabu Search and prove that this approach leads to better economic results than minimizing distances only. Similar problem is considered by Suzuki [23] which develops an approach to the time-constrained, multiple-stop, VRP lowering fuel consumption and pollutants emission by minimizing distance travelled with full load on board. Above two papers indirectly deal with emission minimizing. Kara et al. [14] propose a new cost function based on distance and load of the vehicle for the CVRP (the Energy Minimizing VRP). Apaydin and Gonullu [2] deal with route optimization in solid waste collection in order to decrease diesel emission. They propose using Geographical Information System (GIS) elements such as numerical pathways to problem identification – as it was assumed in this paper.

Another issue is constructing VRP models, especially for time-dependent traffic conditions in cities. The area is well recognized (i.e. state-of-the-art on models [5]) and known methods can be used. Exemplary VRP model with single depot (Cargo Consolidation Centre) in urban conditions was presented by Pyza [22], Jacyna [13] or Jachimowski et al. [10]. Chen, Hsueh and Chang [4] formulate real-time time-dependent VRP with Time Windows with regard to time-dependent travel times. Similar approach is adapted to the problem discussed in this paper. The exact Time Dependant VRP is formulated by Malandraki and Daskin [19] or Figliozzi [6].

VRP models can be solved according to different criteria functions; Jachimowski and Žak [11] formulate VRP with Heterogeneous Customers Demand according to external transportation costs. Jacyna and Klodawski [12] discuss the problem in aspect of transport
co-modality, similarly as źak et al. [25] or [17]. Lewczuk and Wasiak [18] evaluate TSP solving in aspect of cost sharing. Emission of different types as criteria is used in different ways in [2], [3], [14], [15], or [23].

Adduced papers include methods, elements of methods and approaches to TDVRP and CVRP with emission, but do not comprise the approach as it is presented in this paper.

4. FORMAL MODEL

4.1. TRANSPORT NETWORK, INDEXES AND PARAMETERS

Proposed model describes the issue of setting best possible route in time-dependent, urban traffic conditions to minimize CO₂ emission and serve the clients. Let \( S = <G, F_L> \) be the representation of city transport network, where: is a graph of network, \( W \) is a set of nodes, \( L \) is a set of permitted transport links (edges) between nodes, and \( F_L \) is a set of time-dependent functions defining edge transition parameters.

Each link between two nodes representing adjacent crossroads can be composed of a number of sections corresponding to sections that street can be divided into according to differences in parameters values (fig. 1). This results from using Geographic Information System (GIS) to precise modelling, and allows implementing available standard GIS data for different urban areas to use proposed approach in practice.

Each section is represented by pair of edges of opposite directions with known length, number of lines, speed limit, allowed types of vehicles, and variable capacity. The capacity changes in a reaction to traffic conditions and is correlated to area it belongs to. Functions \( F_L \) on the edges must be fitted to represent real movement conditions. All nodes in graph allow turning back.

Fig. 1. The representation of transport link sections in graph

Clients are attributed to selected nodes in graph. The set of clients’ orders is known. The vehicles of different loading capacity can be used. All vehicles start and go back to central depot when empty. Daily work time can’t exceed assumed value. The decision about vehicle type selection, starting hour, and routes must be taken to serve orders with lower CO₂ emission.
Let the following indexes be defined:

- $i, j, k$ – numbers of graph nodes;
- $i, j, k \in W$
- $i$ – client number;
- $i \in C$, $C = \{i : \delta(i) = 1, i \in W\}$
- $s$ – goods vehicle type;
- $s \in S$
- $t$ – time moment;
- $t \in T$
- $m$ – zone number;
- $m \in M$
- $L^m$ – set of edges assigned to $m$-th zone;
- $L^m \subset L$
- $h$ – number of time intervals;
- $h \in H$

and the model parameters:

- $i = 0$ – central depot;
- $h(t)$ – assignation of $t$-th time moment to $h$-th time interval;
- $T_c$ – maximal daily work time set for driver (h);
- $\delta(i)$ – binary value, equal to 1 when $i$-th node has attributed client;
- $v_{ij}^{h(t)}$ – mean velocity between nodes $i$ and $j$ in $h$-th time interval (km/h);
- $v_{ij}^{\text{max}}$ – speed limit between nodes $i$ and $j$ (km/h);
- $d_{ij}$ – distance between nodes $i$ and $j$ (km);
- $\Gamma_{i}^{-}$ – set of predecessors of $i$-th node;
- $\Gamma_{i}$ – set of consequents of $i$-th node;
- $q_i$ – ordered amount of material for client attributed to $i$-th node, if client is not attributed to $i$-th node $q_i = 0$ kg;
- $\tau_i$ – time of loading operations in $i$-th node if client is attributed to the node, if client is not attributed to $i$-th node $\tau_i = 0$ min,
- $Q_s$ – loading capacity of $s$-th type vehicle (kg);
- $w_s$ – curb weight of $s$-th type vehicle (kg);
- $c^s$ – cold start emission of $s$-th type vehicle (g/cold start).

The decision variable is:

- $x_{ij}^s$ – a binary variable equal to 1 if and only if $s$-th type of vehicle is to be present on edge between nodes $i$ and $j$ in $t$-th moment.

### 4.2. CRITERIA FUNCTION AND CONSTRAINS

Minimize:

$$\forall s \in S \quad c^s + \sum_{(i, j) \in L, t \in T} \sum_{i} x_{ij}^{s, t} d_{ij} g^s(v_{ij}^{h(t)}) \Phi(\gamma^t, v_{ij}^{h(t)})$$

where:

$g^s(v_{ij}^{h(t)})$ – the rate of CO$_2$ emission for an unloaded $s$-th type vehicle between nodes $i$ and $j$ in $h$-th time interval (g/km);
\( \Phi(\gamma', v_{lj}^{h(t)}) \) – load correction factor function dependent on \( s \)-th type vehicle, load and velocity.

The rate of CO\(_2\) emission is defined as follows [8]:

\[
g^s(v_{lj}^{h(t)}) = \alpha_1^s + \alpha_2^s v_{lj}^{h(t)} + \alpha_3^s (v_{lj}^{h(t)})^2 + \alpha_4^s (v_{lj}^{h(t)})^3 + \frac{\alpha_5^s}{v_{lj}^{h(t)}} + \frac{\alpha_6^s}{(v_{lj}^{h(t)})^2} + \frac{\alpha_7^s}{(v_{lj}^{h(t)})^3} \tag{2}
\]

where \( \alpha_{1-7}^s \) are the coefficients of emission functions (as given in table 1).

The load correction factor \( \Phi(\gamma', v_{lj}^{h(t)}) \) can be estimated as [8]:

\[
\Phi(\gamma', v_{lj}^{h(t)}) = \beta_1^s + \beta_2^s \gamma' + \beta_3^s (\gamma')^2 + \beta_4^s (\gamma')^3 + \beta_5^s (v_{lj}^{h(t)})^2 + \beta_6^s (v_{lj}^{h(t)})^3 + \frac{\beta_8^s}{v_{lj}^{h(t)}} \tag{3}
\]

where:

\( \beta_{1-8}^s \) – coefficients of the load correction functions (as given in table 2);

\( \gamma_{0, \gamma} \) – the gradients of load (%).

Gradients of load are defined as follows:

\[
\forall i \in W \quad \gamma_{0i} = \frac{m_i + \sum_{k \in C} q_k}{m_s} \cdot 100\% \tag{4}
\]

\[
\forall s \in S \forall t \in T \quad \gamma_{ik} = \sum_{j \in \Gamma_i} \gamma_{ji} x_{ji}^{st} - \frac{q_i}{m_s} \cdot 100\% . \tag{5}
\]

Constrains:

\[
\forall i \in W \quad \forall s \in S \quad \sum_{t \in T} \sum_{j \in \Gamma_i} x_{ji}^{st} - \sum_{t \in T} \sum_{k \in \Gamma_i} x_{ki}^{st} = 0 \tag{6}
\]

\[
\forall (i, j) \in L \quad \forall s \in S \quad \forall h \in H \quad \forall t \in T \quad x_{ji}^{st} v_{lij}^{h(t)} \leq v_{lj}^{max} \tag{7}
\]

\[
\forall s \in S \quad \sum_{i \in W} \sum_{t \in T} x_{ji}^{st} q_i + \sum_{i \in W} \sum_{j \in W \cap j \neq i} \sum_{t \in T} x_{ji}^{st} q_i \leq Q_s \tag{8}
\]

\[
\forall (i, j) \in L^m \quad \forall m \in M \quad \forall t \in T \quad x_{ji}^{st} \leq \text{sgn}(n_{i,s} - \lambda_{v,m}) + 1 \tag{9}
\]

\[
\forall s \in S \quad \sum_{i \in W} \sum_{t \in T} x_{ji}^{st} - \sum_{t \in T} x_{ji}^{st} = 0 \tag{10}
\]
The model is then (1) subject to (6), (7), (8), (9), (10), (11), (12), and (13). Objective function minimizes total emission of CO₂ by s-th type of goods vehicle. Constraint (6) stipulates that each vehicle visiting i-th node (as transitional) will leave it. Constraint (7) ensures that speed limits will not be exceeded. Constraint (8) keeps s-th type vehicle loading capacity not exceeded. Constrain (9) states that vehicles with low emission standard will not enter zones requiring high standards. Constrain (10) ensures that vehicle will came back to the starting point at the end of the route. Constraint (11) ensures that regular and overtime hours lie within the allowable limits. Constrain (12) stipulates that every client will be serviced. Constrain (13) allows disabling selected transport links if (and when) necessary.

The difficulty is in finding coefficients decreasing velocity depending on the time of the day. They can be estimated by empirical measures of traffic in reliable cross-sections of transportation network or can be calculated. Examples of calculation methods can be found in [1]. These coefficients must reflect vehicle’s speed profile, especially instantaneous speed, acceleration, type of fuel, engine power and power torque. Necessary methodology is present in [3], [16], [20], and [24]. At this stage of research coefficients given in [8] are adapted.

5. SOLVING

Proposed NP-hard model demands algorithm finding sufficient solution in acceptable time. A two-step heuristics based on genetic programming for frame searching, and A-star algorithm for pathfinding was developed. A-star was modified to meet time-dependant traffic conditions in urban areas. The general idea is to solve classic TSP deprived of constrain limiting visiting nodes only once. Efficient traversable paths between nodes are plotted by modified A-star algorithm with regard to CO₂ emission level and time.

A-star algorithm (Hart, Nilsson and Raphael [7]) searches for least-cost path by evaluating only one cost measure at the time. CO₂ emission level is subjected to traffic conditions and differs when speed and acceleration are changing in time, so A-star was adapted to deal with that problem. The distance-plus-cost heuristic function in A-star is a
sum of the path-cost function from starting to current node, and admissible heuristic estimation of cost of traveling from current to finish node. The path-cost function, unlike the classic formulation, calculates CO₂ emission dependant on traffic conditions, loading and vehicle type in current time interval $h(t)$. It also calculates the time of previous travel to find out in which time interval the next stage will start, so traffic constrains for next stage can be found.

The heuristic estimation of the cost of remaining travel must consider time-dependant conditions. Firstly a straight line connecting the coordinates of current node and finish node is plotted. Registered intersections with graph edges cut plotted line into segments that are to be temporary edges used to estimate cost of remaining travel. Transitions costs for these new edges are adopted from genuine edges with rule that new edge has a cost of edge with which it crosses at the end (fig. 2). Transition costs change in time, so attributing cost from genuine edges must take into account the time of travel through new edges. This time is estimated with using information about area it belongs to, actual time and coefficient of movement resistance. Finally, using modified A-star algorithm results in calculating for each considered pair of nodes the total CO₂ emission and travel time. Emission will be counted by criteria function and time will be an input parameter for next pair of nodes.

TDVRP based on relaxed TSP can be solved efficiently for static conditions. Taking into account variable conditions and the fact that total cost is not expressed in natural measure like time or distance the genetic programming was applied to solve the problem on the second stage of algorithm. Chromosome encodes the sequence of clients to be visited. Two types of crossover operators are used: 1/ one cut-point, and 2/ PMX with two cut-points. Four different policies of selecting pairs of parents were considered: 1/ sequentially best with best, 2/ sequentially with random, 3/ sequentially best with worst, and 4/ random with random. The mutation coefficient changes during calculation. At the beginning it keeps reference value, and increases together with the calculation progress.
Thanks to that in the first stage of solving, recombination has a dominant role, while in the second stage extensive mutation of best individuals usually leads to improvement. Experiments were carried for fixed number of generations (3 000 to 10 000) embracing constant number of individuals (30 to 70). The new generation was composed of 10 to 20% of best parents and 50 to 60% of best fitted offspring. The rest was randomly selected.

**6. CALCULATION EXAMPLE**

At this stage of research only preliminary experiments were carried. Exemplary transportation network represented by graph of about 4 700 edges and 1 800 nodes was adopted. City is divided into centre area and outside area. Flow capacity is estimated for 27 time intervals, 20 minutes each, starting from 5 am. 21 clients are placed in selected nodes. Loading operations in client’s points take 4 to 10 minutes. This pattern is characteristic for small FMCG shops.

The classification of goods vehicles was adapted from [8] and it is presented in table 1 together with necessary coefficients that meet formulas (2) and (3).

**Table 1**

<table>
<thead>
<tr>
<th>Vehicle type $s$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$c_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 – 7.5</td>
<td>110</td>
<td>0</td>
<td>0</td>
<td>0.000375</td>
<td>8702</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7.5 – 16.0</td>
<td>871</td>
<td>-16.0</td>
<td>0.143</td>
<td>0</td>
<td>0</td>
<td>32031</td>
<td>0</td>
</tr>
<tr>
<td>16.0 – 32.0</td>
<td>765</td>
<td>-7.04</td>
<td>0</td>
<td>0.000632</td>
<td>8334</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>32.0 – 40.0</td>
<td>1576</td>
<td>-17.6</td>
<td>0</td>
<td>0.00117</td>
<td>36067</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Vehicle type $s$</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
<th>$\beta_5$</th>
<th>$\beta_6$</th>
<th>$\beta_7$</th>
<th>$\beta_8$</th>
</tr>
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<tr>
<td>3.5 – 7.5</td>
<td>1.27</td>
<td>0.0614</td>
<td>0</td>
<td>-0.00110</td>
<td>-0.00235</td>
<td>0</td>
<td>0</td>
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<td>7.5 – 16.0</td>
<td>1.26</td>
<td>0.0790</td>
<td>0</td>
<td>-0.00109</td>
<td>0</td>
<td>-2.03E-7</td>
<td>-1.14</td>
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<tr>
<td>16.0 – 32.0</td>
<td>1.27</td>
<td>0.0882</td>
<td>0</td>
<td>-0.00101</td>
<td>0</td>
<td>0</td>
<td>-0.483</td>
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</tr>
<tr>
<td>32.0 – 40.0</td>
<td>1.43</td>
<td>0.121</td>
<td>0</td>
<td>-0.00125</td>
<td>0</td>
<td>0</td>
<td>-0.916</td>
<td></td>
</tr>
</tbody>
</table>

Set of experiments was carried. For exemplary data: 7 000 generations, population of 40 specimens, crossover made sequentially with random with probability of 0.85, and mutation probability of 0.05, mutation change coefficient 1.9 and selecting 20% of best parents and 60% of best offspring (rest is selected randomly) the total emission of CO2 was 20920 g at 225.45 min (3.5 – 7.5 tons vehicle). The genetic algorithm suitting function was convergent, but the best suitable solution was found during extensive mutation. The path founded for exemplary data is plotted on fig. 3. Appropriate comments are added.
7. CONCLUSIONS AND FURTHER RESEARCH

Presented research is the introduction to wider program aiming in setting guidelines for designing pro-ecological transport systems. Using heuristic tools for solving class of VRP is considered effective and leads to improvement in cost efficiency or emission levels. Correlating problems of distance minimization with emission minimization allows developing wide range of tools to improve living conditions in urban areas.

Further research will take into account estimation of emission of other harmful exhaust components, applying Euro Standard for engines into model, multi TSP as an expansion of proposed approach, continuous time-dependant functions describing transitions characteristics, and additional modifications of presented problem.

In this respect, the estimation and modelling of CO\textsubscript{2} can be a powerful tool for air quality managers and environmentalists in order to examine the impact of different transport plans.

Fig. 3. Solution founded for exemplary data
Acknowledgment

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Bibliografia


TRASOWANIE POJAZDÓW Z UWZGLĘDNIENIEM OGRANICZEŃ ŁADOWOŚCI
ZE WZGLĘDU NA EMISJĘ CO₂ W WARUNKACH MIEJSKICH

Streszczenie: W artykule podjęto zagadnienie trasowania pojazdów dostawczych w warunkach miejskich ze względu na emisję CO₂ i czas przejazdu. Warunki ruchowe charakterystyczne dla obszarów miejskich i ich zależność od czasu zostały zdefiniowane. Kwestie ekologiczne emisji CO₂ w warunkach miejskich zostały wykorzystane do skonstruowania modelu matematycznego dla zależnego od czasu problemu trasowania pojazdów z określoną masą ładunkową, którego funkcja kryterium zakłada minimalizację emisji dwutlenku węgla. W modelu ujęto charakterystyki infrastruktury transportowej obszarów zurbanizowanych, natężenie ruchu zależne od pory dnia i różne typy pojazdów dobierane w zależności od wielkości realizowanego zadania. Model został zaimplementowany w rzeczywistej sieci transportowej. Dla danych projektowych wykonano obliczenia z wykorzystaniem dwustopniowej heurystyki bazującej na zmodyfikowanym algorytmie A* oraz programowaniu genetycznym. Otrzymane rezultaty, kierunki przyszłych badań oraz potencjalne zastosowania zostały zaprezentowane i omówione.

Słowa kluczowe: emisja CO₂, trasowanie w warunkach zmiennych w czasie, CVRP, logistyka miejska