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Anna Kwasiborska
Warsaw University of Technology

THE PROBLEM OF SEQUENCING LANDING AIRCRAFT

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Abstract: Planning of aircraft traffic in airspace has a great impact on flight safety and costs related to air transport. Improper air traffic planning causes unnecessary delays, which not only discourage passengers from using services of a given air carrier, but also significantly increases fuel costs.
This article aims to consider aspect of air traffic optimization in approach area in proximity of an airport. Initially, real problem of landing sequencing will be presented. Next, approach procedure modelling will be defined, so it is as close to real approach procedure as possible. The landing aircraft sequencing problem was solved with created integer model. It allows to combine the setting of landing sequence with geometric carrying of flights. However, when compared to existing papers, this article will focus not only on aircraft sequencing, but also will include its implementation. Such approach makes it possible to follow safety standards required during aircraft landing approach. Objective function is presented to find the best solution to the problem under consideration.
Keywords: scheduling of landings aircraft, aircraft landing sequencing, optimized air traffic management

1. INTRODUCTION

Over the last years there has been significant increase of air traffic. Air Traffic Control services see more and more difficulties with operating ever increasing amount of aircraft. This is the reason why the need for tactic planning of air space traffic appeared. In the peak rush hours the throughput often get close to, or even exceed, maximum landing capacity. That is why air traffic planning is necessary on earlier stage. With information when subsequent planes will be approaching for landing, it is possible to coordinate their flights to streamline the traffic. The references contain numerous examples of tactical planning of air traffic.

In the paper [2], Beasley et al. consider the problem of landing sequencing with integer programming. Objective function is studied, which minimizes costs of landings depending on whether aircraft arrived too early or too late compared to planned time, and on the time difference. Cost function examined in the paper is linear in parts.

In paper [7] Soomer et al. present another integer problem with partially linear cost function. Scheduled times of aircraft landings are given, and on this basis authors attempt to sequence landings to minimize the costs. Air lines preferences are also considered, which noticeably affect solution found. Balakrishnan et al. [1] show different approach to the problem. Aircraft landing base schedule is given. With defined maximum number of changes in the aircraft sequence, authors tried to modify the schedule to obtain the best possible solution.

2. PROBLEM DESCRIPTION

One of the APP (Approach Control) tasks is efficient guiding of aircraft within distance of 100–150 km from the airport, until landing operation. Research problem aims to optimise air traffic in the airport approach area, and study was performed on the example of the airport in Warsaw. Approach area is defined as air space section operated by approach control services. In order to regulate air traffic, STAR (Standard Terminal Arrival Route) procedures have been defined, and shown on fig. 1 for runway RWY 33.

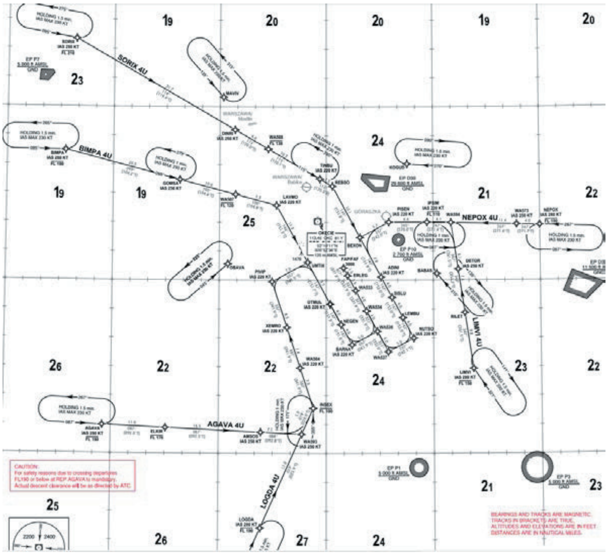

Fig. 1. Arrival route for RWY 33
Sources: AIP Polska

Possible aircraft routes around the airport have been defined in directed graph. The vertices are the points inside approach area (inlet gates) and edges correspond to the connections between the points, in accordance with aircraft routes direction. Edge weights are distances between given points. The graph is constructed to describe key dependencies shown on the map in the best way possible. However, aircraft routes do not have to precisely overlap with the routes on the map, so the graph contains edges whose counterparts may be missing on the map.

Aircraft approach to the airport is also strictly standardized. The following conditions must be met:

– aircraft travel at proper speeds, depending on the place it currently is;
– time or distance separation between aircraft is obligatory, depending on the current position;
– each aircraft has its own weight class, and it may additionally affect separation between different aircraft;
– aircraft do not have to move strictly along air routes on the map, air traffic control may request moving to route between points without common edge (i.e. to use shortcuts).

The problem may seem to be trivial, but it certainly is not.

2.1. MODEL DESCRIPTION

The model aims to describe aircraft traffic management in the airport approach area. We assume that possible aircraft positions will be described by diagram shown on fig. 2.

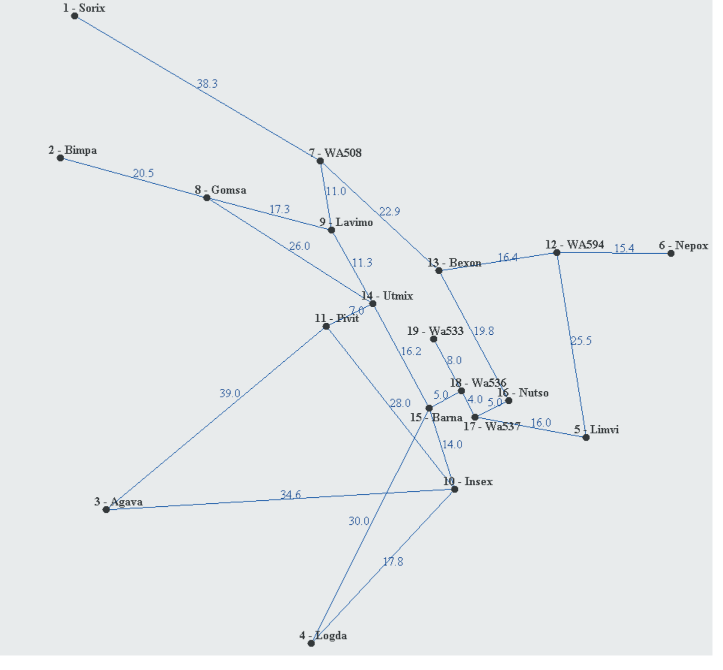

Fig. 2. Approach area scheme
Sources: own elaboration

This is an approximation of the real map, with addition of few extra paths than aircraft may be transferred to. In the task the following assumptions have been made:

1. In the time from entering air space around the airport until landing (reaching the point with the highest number), aircraft may take positions only on the points with numbers or on the edges that connect the points.
2. Each aircraft may be in one place at the same time.
3. Each aircraft entering the approach area is given unique number, arrival time in minutes (must be at least 1), point number that is initial in the airport approach path.
4. Air traffic follows edges in one direction only, i.e. from the points with lower number towards points with greater number.
5. Numbers shown at the edges describe their lengths in nautical miles (1 NM is 1852 metres).
6. Aircraft move at uniform motion along whole length of an edge.
7. Allowed aircraft speed is 220 NM/h on all edges with exception of the edge that connects the last two points, where the allowed speed is 160 NM/h. We assume that aircraft real speed may differ from allowed speed by 10% maximally.
8. Aircraft at approach must be separated with certain distance, on the edges starting from vertices 1–12 the distance is 0 NM. We assume that aircraft are at different altitudes (exact altitudes are irrelevant in this model). On edges starting from points 13–18 separation is 5 NM and on the last edge that connects points 18 and 19 the separation is 3 NM. On these segments aircraft must be so low that they cannot take altitudes that would be sufficiently different. It is assumed that an edge starts at its starting vertex and does not include its end vertex.
9. Aircraft cannot overtake along edges where separation is required.

The model does not include weight separation. It was assumed that analysed group of aircraft is of the same type. However, it is very easy to modify the model described below to change this assumption.

The following data is assumed:

Sets:

PWejsc – set of initial points for aircraft approaching the airport.

P – set of all points of the graph.

PDalsze – set of points without starting points.

PBezKonca – set of points excluding the last one.

S – set of all aircraft to be scheduled in air traffic.

SS – set of all pairs of different aircraft.

PSep – set of points with non-zero separation.

PPpolaczone – set of pairs of points that are connected with common edge, where the first point of the pair is the starting point of the edge.

PunktyDalej[PBezKoncza] – family of sets dependent on the set *PBezKonca*. For each point *p* in the set *PBezKonca, PunktyDalej[p]* denotes set of such vertices, for which vertices exists edge with *p*.

PunktyWczesniej[PDalsze] – family of sets dependent on the set *PDalsze*. For each point *p* in the set *PDalsze*, *PunktyWczesniej[p]* denotes set of points, for which points there is an edge that ends in *p*.

PPSep – subset of the point pairs set *PPpolaczone*, such that both points have non-zero separation.
PunktyDalejOMniejszymNumerze[PPpolaczone] – family of sets dependent on the edge, for each edge ‹p1; p2›, *PunktyDalejOMniejszymNumerze[p1; p2]* is a set of points *p3*, such that there is an edge ‹p1; p3› and *p3 < p2* (in the sense of the point number).

Parameters:
Przylot[S] – time of entering air space around the airport for each aircraft.
PW[S] – point where aircraft enters approach area.
Ostatni – the last point that denotes airport.
inf – very large numerical constant, significantly greater than distances on the air map (e.g. 1000000). It will be used to simplify notifications of expressions.
V [PDalsze] – allowed speed at points. Speed allowed on edges takes value equal to allowed speed of the next point to visit.
sep[PSep] – separation given in miles, depending on the point the aircraft is currently at.
Dist[P x P] – matrix that describes distances between points. If there is no edge between the points, the distance is equal to *inf.*

Variables:
T[S x P] – real variable that indicates when aircraft should be at a given point.
If aircraft never visited the point, the variable equals 0 for that point.
czy[S x P] – binary variable that indicates if aircraft should be at a given point, if yes it is equal to 1, in other case it is equal to 0.
pierwszy[SS x PSep] – binary variable that indicates, which aircraft of the pair was first at a given point. It is equal to 1 if the first aircraft of the pair was sooner, and 0 in the opposite case.

Restrictions:
1. Basic restrictions, linking variables.

$$\forall s,p \in S\ x\ P : T\,[s,p] \le \inf \cdot\ czy[s,p] \tag{1}$$

If aircraft does not go through a given point, then restriction enforces that the time of flight through this point is equal to 0.

$$\forall s,p \in S\ x\ P : T\,[s,p] \ge czy[s,p] \tag{2}$$

Restriction enforcing that arrival time is not equal to 0, if the aircraft went through given point. This means that the time must be defined.

$$\forall s,pp \in S\ x\ PBezKoncza : czy[s,pp] \le \sum_{p \in PunktyDalej[pp]} czy[s,p] \tag{3}$$

Restriction enforcing that presence of aircraft in a certain point also means that the aircraft must have been also in at least one of the point's successors.

$$\forall s, pp \in S \; x \; PDalsze : czy[s, pp] \leq \sum_{p \in PunktyWczesniej[pp]} czy[s, p] \tag{4}$$

Restriction enforcing that presence of aircraft in a certain point also means that the aircraft must have been also in at least one of the point's predecessors.

$$\forall s \in S : czy[s, 15] + czy[s, 17] = 1 \tag{5}$$

Equation allows arrival at the last-but-one point only from one direction.

2. Inequations that describe behaviour in entry points.

$$\forall s \in S : T[s, PW[s]] = Przylot[s] \tag{6}$$

Restriction enforcing that variable *T* for each aircraft for its entry point will take the time of arrival to the point as indicated in the data.

$$\forall s \in S : czy\big[s, PW[s]\big] = 1 \tag{7}$$

Restriction enforcing that for all aircraft variable *czy* takes value *1* in the starting point, which is the same as the point in the data.

$$\forall s \in S : \textstyle\sum_{p \in Pwejsc} czy[s, p] = 1 \tag{8}$$

Restriction enforcing that aircraft may appear only in one entry point.

3. Restrictions that describe behaviour in the end point.

$$\forall s \in S : czy[s, Ostatni] = 1 \tag{9}$$

Equation that ensures that each aircraft will eventually reach the end point, i.e. it will land.

4. Restrictions that enforce aircraft to observe allowed speed.

$$\begin{aligned} \forall s, p1, p2 \in S \; x \; PPpolaczone : (T[s, p2] - T[s, p1]) \cdot V[p2] \cdot 1.1 \\ \geq Dist[p1, p2] \cdot 60 - (1 - czy[s, p1]) \cdot inf - (1 - czy[s, p2]) \cdot inf \end{aligned} \tag{10}$$

The condition enforce that the aircraft speed between two connected points is not greater than maximal. If the aircraft has not been in one of the points, then inequation becomes trivial and it is always true. Let's note, that if both vertices were visited, then we obtain inequation equivalent to the inequation that checks if the aircraft speed is not greater than maximum speed (notice that constant *inf* should be great enough that dividing it by constants from this inequation should not bring *inf* to order of magnitude of other variables).

$$\forall s, p1, p2 \in S\ x\ PPpolaczone : (T[s,p2] - T[s,p1]) \cdot V[p2] \cdot 0.9 \\ \leq Dist[p1,p2] \cdot 60 + (1 - czy[s,p1]) \cdot inf + (1 - czy[s,p2]) \cdot inf \\ + \sum_{p3 \in PunktyDalejOMniejszymNumerze[p1,p2]} (czy[s,p3] \cdot inf) \tag{11}$$

Inequation that makes the aircraft speed between two connected points not lower than minimum. If the aircraft has not been in one of the points, then inequation becomes trivial and it is always true. If the aircraft has been in a given point, but got there along another path (i.e. that includes point with smaller number), the situation gets trivial too. In other cases we obtain inequation equivalent to the one that checks, if the aircraft speed is not lower than minimum speed.

5. Conditions that enforce appropriate separation between aircraft.

$$\forall s1, s2, p1, p2 \in SSxPPSepDist[p1,p2] \cdot (T[s2,p1] - T[s1,p1]) \\ \geq sep[p1] \cdot (T[s1,p2] - T[s1,p1]) - (1 - czy[s1,p1]) \cdot inf \\ - (1 - czy[s2,p1]) \cdot inf - (1 - czy[s1,p2]) \cdot inf \\ - pierwszy[s1,s2,p1] \cdot inf \tag{12}$$

Restriction that keeps appropriate distances at points with non-zero separation. It checks if the distance has been maintained for each point and between each pair of aircraft. If one of the aircraft has not visited a given point, or the first aircraft of the pair has been at the point earlier, then the inequation is always true. In the opposite case there must be appropriate separation.

$$\forall s1, s2, p1, p2 \in SSxPPSep: (Dist[p1,p2] - sep[p2]) \cdot (T[s1,p2] - T[s1,p1] \\ \geq Dist[p1,p2] \cdot (T[s2,p2] - T[s1,p1]) - (1 - czy[s1,p1]) \cdot inf \\ - (1 - czy[s1,p2]) \cdot inf - (1 - czy[s2,p2]) \cdot inf \\ - pierwszy[s2,s1,p1] \cdot inf \tag{13}$$

The second condition that keeps appropriate separations between aircraft.

$$\forall s1, s2, p \in SSxPSep : pierwszy[s1,s2,p] + pierwszy[s2,s1,p] = 1 \tag{14}$$

Equation that makes the first variable behave naturally. For each aircraft pair and any given point only one machine could be the first at the point. In case only one aircraft (or none) visited given point, the variable may take any value, because it doesn't affect further considerations anyway.

Objective function:

$$Minimize: \sum_{s \in S} T[s, Ostatni]$$

Minimizes total time of aircraft landings.

2.1.1. Separations between aircraft

It must be shown, that conditions (12) and (13) really enforce appropriate separations between aircraft. We must notice, that we must take care of separation only when two aircraft have a route section in common (including at least one vertex). Firstly, let's consider the below described situation.

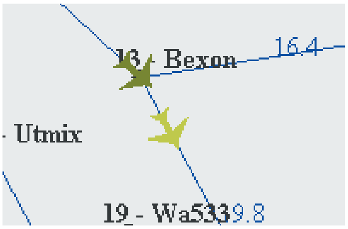

Fig. 3. Example of aircraft moves and consideration of separation
Sources: own elaboration

Let the aircraft at front be *S1*, and subsequent aircraft be *S2*. *S2* is located at a certain point, that S1 must have also flown through. Let *T1* be the time of taking the point by *S1*, and by analogy *T2* is the time for *S2* (obviously, $T1 < T2$). Let T1' be the time, when *S1* takes subsequent point (i.e. the point it is currently heading to). Let's notice, that in the time span *T1* to *T2*, the first aircraft must have flown at least separation distance (mark it as *Sep*), therefore:

$$(T2 - T1) \cdot V1 \geq Sep \tag{14}$$

Where *V*1 is the speed of the first aircraft (we assume that the speed is constant along the edges).
We notice, that:

$$V1 = \frac{Dist}{T1` - T1} \tag{15}$$

where *Dist* is the distance reflecting the edge length, on which *S1* is located. When these two conditions are combined, we obtain inequation responsible for separation in such cases. Of course it is necessary to add expressions that make the inequation trivial, when any of the aircraft does not travel through a point or the aircraft sequence is different. After such operation we obtain condition (12).

2.1.2. Being at one place

The model creates obligation, that from each point aircraft must go to at least one subsequent point, and that each point may be reached from at least one preceding point (obviously one of the assumptions is not met for start point and end point). When we add assumption, that only one start point may be taken, we obtain condition, that aircraft being in two places at the same time requires existence of two paths of similar lengths, which could be picked at the same time. The only section of the approach area that makes such situation potentially possible is the section from point WA508 to point WA536, all other ramifications set up the route unambiguously.

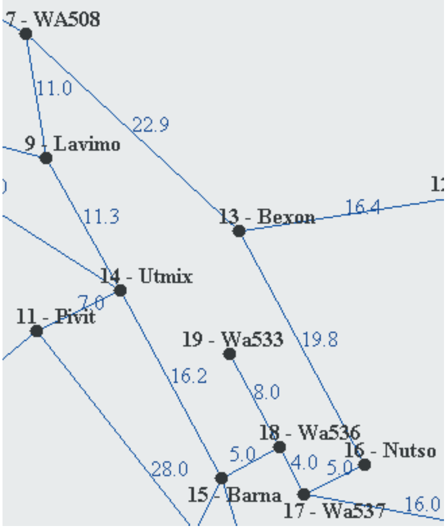

Fig. 4. Problematic route section
Sources: own elaboration

We will consider all ramifications to show this. Ramifications in LIMVI, INSEX direct aircraft into two paths, that differ in length significantly. It is easy to show, that even at the greatest speed along the longer path, the time of taking the first common point will be greater than time of taking this point with the lowest speed on the shorter path. Ramifications in LOGDA, AGAVA and GOMSA may at most cause missing one point on the route. However, variable *czy* for this point defines unambiguously, which route has been picked. So the ramification at WA508 is the only place, where it is possible to „split" the aircraft.
Let's note however, that limitation (5) enforces that approach starts from one side only. Therefore the situation, when the aircraft is in two places at the same time, is impossible.

2.1.3. Greedy algorithm

Greedy algorithm always chooses locally optimal solution. In this case it will make aircraft to approach at maximum possible speed, and in conflict situations it will ensure speed reduction for aircraft that would enter common route as the second one of two aircraft. It is worth asking, whether greedy algorithm would find optimal solution to the problem. We will consider below described example.

The following section of air space map is considered. Let's assume that remaining edges are full with aircraft and it is impossible to send another machine there (fig. 5).

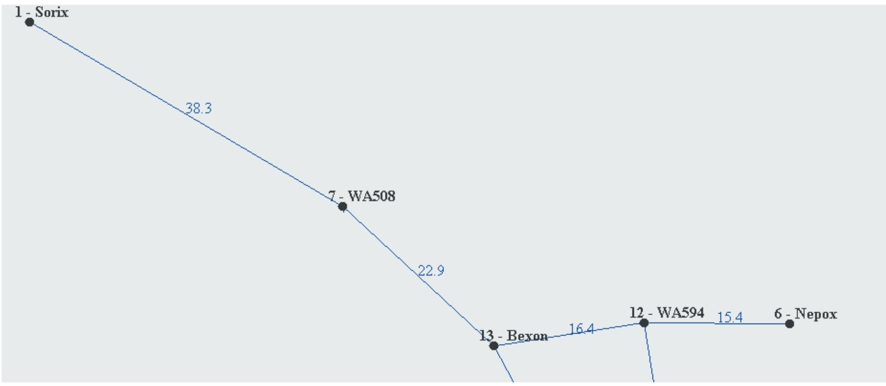

Fig. 5. Section of approach area
Sources: own elaboration

Bexon point has non-zero separation between aircraft, whereas in other points in the map (fig. 5) the separation is not required. We want to plan aircraft traffic, and we know that:

Table 1

Identified times at selected points of entry

No aircraft	Entry time	Entry point	T_{min}	T_{max}
1	1	1	16,17	19,55
2	8,31	6	16,20	17,94
3	8,31	6	16,20	17,94

Sources: own elaboration

where T_{min} and T_{max} are minimum and maximum time respectively, that the aircraft needs to reach point 13 (given with certain accuracy). Starting from Bexon point, separation of 5 NM is required and assuming maximum speed after taking the point (244 NM/h) it means time separation of at least 1,24 between aircraft that intend to fly through the point. If we send through this point the aircraft 1 at first (which can be there soonest), and next aircraft 2 (or 3), then in the time range 16,17 to 18,65 no other aircraft can go through this point. So there is no way for the last aircraft to go.

However, if we send aircraft 2, then 3, and aircraft 1 as the last one, then all aircraft can go through the point. Now we see that sending first aircraft that can be soonest at a given place is not a good solution to the problem.

Reason why this example cannot be solved by greedy algorithm is the difference in length of two different routes. On the longer route we can delay reaching the point in conflict more than on the shorter route. That is why solving the problem requires analysis of whole approach area, not only which aircraft can get to a point sooner.

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PROBLEMATYKA SEKWENCJONOWANIA SAMOLOTÓW LĄDUJĄCYCH

Streszczenie: Planowanie ruchu samolotów w przestrzeni powietrznej ma ogromny wpływ na bezpieczeństwo lotu samolotem oraz koszty związane z transportem. Złe rozplanowanie ruchu lotniczego powoduje niepotrzebne opóźnienia, które nie tylko zniechęcają pasażerów do danego przewoźnika lotniczego, ale także znacznie zwiększają koszty paliwa.

Celem niniejszego artykułu jest poruszenie zagadnienia optymalizacji ruchu samolotów w obszarze zbliżania wokół lotniska. Na początku zostanie przedstawiony rzeczywisty problem dotyczący szeregowania lądowań. Następnie będzie prowadzone modelowanie procedury zbliżania tak, aby była jak najbardziej przybliżona do tej rzeczywistej. Stworzony model całkowitoliczbowy posłużył do rozwiązania problemu szeregowania samolotów lądujących. Uwzględnia on połączenie ustawiania kolejności lądowań z geometryczną realizacją lotów. Jednak w porównaniu do poprzednich prac, nie będzie on skupiał się tylko na szeregowaniu samolotów, ale również będzie uwzględniał jego realizację. Dzięki takiemu podejściu możliwe jest zapewnienie norm bezpieczeństwa wymagane podczas zbliżania się samolotu do lądowania. Przedstawiona została funkcja celu, aby znaleźć najlepsze rozwiązanie do rozważanego problemu.

Słowa kluczowe: szeregowanie lądowań samolotów, sekwencjonowanie samolotów lądujących, optymalizacja zarządzania ruchem lotniczym