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MODIFIED STRIP METHOD SPECIFICATION
FOR WHEEL / RAIL CONTACT STRESS EVALUATION

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Abstract: The Strip Method is often used for rail/wheel contact area and contact normal stress evaluation. The paper deals with the computational time saving procedure when the computation accuracy is guaranteed. We included the local coordinate system with a presupposed semi-circular course of the normal stress for the purpose of the integral that is necessary for deformation in the middle of individual strips evaluation. The integral is solvable analytically. The input parameters for individual parts of splines and individual strips expression are possible to insert after analytical solution.

Keywords: wheel/rail contact, contact stress evaluation, modified Strip method, optimized computation procedure, Contact-TANG, Contact-NORM, FASTSIM

1. STRIP METHOD

The Strip Method presupposes quasi-static rolling [3]. The principal idea of the theory is to take into consideration slim contact areas in the y-direction. In Fig. 1 there are two bodies in contact. In fact the geometrical parameters (of railway wheel and rail) should be similar to the reality. The deformation zones are of the similar shapes too. In spite of this fact the parameters (the displacements \( w_1 \) and \( w_2 \)) in Fig. 1 are rather different for better understanding of the theory.

In fact, the contact area should be a plane (parallel with the x-y plane).

The method presupposes the existence of two rotating bodies 1 and 2 with surfaces \( S1 \) and \( S2 \). The bodies touch in the point 0, which is at the same time the beginning of their spatial coordinate systems. The axes \( x \) and \( y \) determine the horizontal base. We will mark the horizontal coordinate as the \( z \) – axis. If there is no influence of a normal force \( Q \), then there exclusively exists geometrical binding between the bodies.

If the bodies are pressed against each other by the normal force \( Q \), a deformation and a contact area \( \Omega \) instead of a contact point arises between the bodies.
The geometrical profile shape of the first body surface will be marked \( f_1(x, y) \), the geometrical profile shape of the second body surface will be marked \( f_2(x,y) \).

The elastic displacement in the z-axis direction caused by the deformation of the first body surface will be marked \( w_1(x, y) \), the displacement in the z-axis direction caused by the deformation of the second body surface will be marked \( w_2(x, y) \).

The displacement of bodies centers against each other in the axis-z direction will be marked \( d \).

The perpendicular distance between the points of the deformed bodies surfaces will be marked \( \delta(x,y) \).

\[ \text{2. NORMAL STRESS} \]

The stress evaluation over the \( k^{th} \) strip (Fig. 2) is approximated in a standard way [3]:

\[
p_k(x, y) = p_{0k} \cdot \sqrt{1 - \left( \frac{x}{x_{sk}} \right)^2} \tag{1}
\]

where:

- \( p_k(x, y) \) normal stress in the \( x \) position,
- \( p_{0k} \) maximum normal stress,
- \( x_{sk} \) half length of the \( k^{th} \) strip.
We aimed our interest at increasing the computational effort. To obtain the requested results, deformations and stresses in the middle of strips, we utilized the procedures introduced in the next text.

Fig. 2. Contact patch distribution into the strips, stress distribution over individual strips

2.1. TIME OPTIMIZED COMPUTATIONAL PROCEDURE FOR CONTACT STRESSES ASSESSMENT

The equation system assembly is necessary for the stress evaluation in the middle of strips [8]:

\[
[M] \cdot \{\rho^{(0)}\} = \{w^{(0)}\}
\]

where:
- \([M]\) influence coefficients matrix,
- \(\{\rho^{(0)}\}\) normal stresses in the middle of strips vector,
- \(\{w^{(0)}\}\) in the middle of strips strains vector.

The \([M]\) matrix elements are of values:

\[
M_{k,s} = H \cdot I_{k,s}^{(0)}
\]

where:

\[
H = \frac{1}{\pi} \left( \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2} \right)
\]

where:
- \(\mu_i\) Poisson’s ratio of a first body,
\( \mu_2 \) Poisson’s ratio of a second body,
\( E_1 \) modulus of stiffness of a first body,
\( E_2 \) modulus of stiffness of a second body.

The vector elements are:

\[
\mathbf{w}^{(0)}_k = z_{ik} - z_{ik} - D
\]

where:
\( \mathbf{w}^{(0)}_k \) deformation in the middle of \( k^{th} \) strip,
\( z_{ik} \) \( z \)-coordinate of a first body in the middle of \( k^{th} \) strip,
\( z_{2k} \) \( z \)-coordinate of second body in the middle of \( k^{th} \) strip,
\( D \) straightening up of bodies.

The aim is to compute the value \( I^{(0)}_{k,s} \) in accordance with relation below:

\[
I^{(0)}_{k,s} = 2 \cdot \int_{y_s}^{y_k} \int_{y_s}^{y_k} \frac{1 - \left( \frac{x}{x_{ik}} \right)^2}{x^2 + (y - y_s)^2} \cdot dy \cdot dx
\]

when:

\[
y_{ik} = y_k - y_s
\]

\[
y_{ik} = y_k + y_s
\]

where:
\( x_{ik} \) half length of the \( k^{th} \) strip,
\( y_k \) \( y \)-coordinate of \( k^{th} \) strip,
\( y_s \) \( y \)-coordinate of \( s^{th} \) strip,
\( y_s \) half width of strips.

This integral can be modified into one-dimension integral in the form:

\[
I^{(0)}_{k,s} = -\frac{2}{x_{ik}} \cdot \int_{y_s}^{y_k} \sqrt{x_{ik}^2 - x^2} \cdot \left( \sqrt{x^2 + (y - y_s)^2} + y_s - y_{ik} \right) \cdot \left( \sqrt{x^2 + (y_s - y_{ik})^2} - y_s + y_{ik} \right) \cdot dx
\]

It is unavoidable to solve the integral in the relation (9) numerically, because it has no analytical solution. This computation is very time consuming for separate strips in the relationship to other strips.
This is the reason for the new procedure development that is not dependent on a real strip length. We presuppose that a strip is of one unit length. Description of individual parameters in the \( x, y \) coordinate system is in Fig. 3 and a description of individual parameters in the \( u, v \) coordinate system is in Fig. 4.

For the value \( I^{(0)}_{k,s} \) the relation is valid:

\[
I^{(0)}_{k,s} = i^{(0)}_{k,s} \cdot x_{dk}
\]

(10)

where:

\[
b_k = \frac{y_k}{x_{dk}}
\]

(11)

The relationship for the \( I^{(0)}_{k,s} \) computation is valid:

\[
i^{(0)}_{k,s} = 2 \cdot \int_{0}^{1} \int_{-1}^{1} \frac{1-u^2}{u^2 + \left( \frac{1-s+|k| \cdot v}{b_k} \right)^2} \cdot dv \cdot du
\]

(12)

The value: “\( i \)” of this integral or value: “\( E \)” of decimal logarithm of integral are depicted in graphs in Figs. 5 - 10.
Fig. 5. The integral value “i” in dependence on Log(Bk) on condition that |s - k| = 0

Fig. 6. The integral decimal logarithm value “E” in dependence on Log(Bk), on condition that |s - k| = 0

Fig. 7. The integral value “i” in dependence on Log(Bk) on condition that |s - k| = <2;8>

Fig. 8. The integral value “I” in dependence on Log(Bk) on condition that |s - k| = <2;8>

Fig. 9. The integral value “i” in dependence on Log(Bk) on condition that |s - k| = <10;40>

Fig. 10. The value of integral decimal logarithm in dependence on Log(Bk) on condition that |s - k| = <10;40>
The value of an integral decimal logarithm “E” can be expressed via the polynomial of 6-th grade:

\[ E = \left( \left( \left( \left( \left( \left( \left( a_0 \cdot L + a_6 \right) \cdot L + a_4 \right) \cdot L + a_2 \right) \cdot L + a_0 \right) \cdot L + a_2 \right) \cdot L + a_0 \right) \cdot L + a_4 \right) \cdot L + a_2 \right) \cdot L + a_0 \]  

(13)

where:

\[ L = \log \left( b_k \right) \]  

(14)

Polynomial coefficients \( a_6 \) to \( a_0 \)

| \(| s - k |  \| a_6 \| a_5 \| a_4 \| a_3 \| a_2 \| a_1 \| a_0 |
|---|---|---|---|---|---|---|
| 0 | -0.002505 | -0.001497 | 0.022573 | 0.009238 | -0.16037 | 0.467393 | 0.483852 |
| 2 | -0.007560 | -0.026714 | 0.015249 | 0.105083 | -0.14821 | 0.079591 | -0.08251 |
| 4 | 0.003839 | -0.008085 | -0.029310 | 0.068803 | -0.04814 | 0.015369 | -0.39959 |
| 6 | 0.007608 | 0.004252 | 0.037180 | 0.037078 | -0.0153 | 0.005056 | -0.5794 |
| 8 | 0.007902 | 0.009316 | -0.035130 | 0.019712 | -0.0027 | 0.002233 | -0.7058 |
| 10 | 0.007030 | 0.010980 | -0.030570 | 0.010126 | 0.00236 | 0.001448 | -0.80338 |
| 12 | 0.005830 | 0.011066 | -0.025740 | 0.004737 | 0.004248 | 0.001116 | -0.88289 |
| 14 | 0.004628 | 0.010462 | -0.021300 | 0.001656 | 0.004707 | 0.000973 | -0.95 |
| 16 | 0.003549 | 0.009613 | -0.017440 | -0.000140 | 0.004506 | 0.000906 | -1.00807 |
| 18 | 0.002611 | 0.008678 | -0.014150 | -0.001150 | 0.004019 | 0.000859 | -1.05925 |
| 20 | 0.001806 | 0.007735 | -0.011380 | -0.001650 | 0.003431 | 0.000813 | -1.10502 |
| 22 | 0.001127 | 0.006837 | -0.009070 | -0.001860 | 0.002834 | 0.000765 | -1.1464 |
| 24 | 0.000534 | 0.005942 | -0.007140 | -0.001820 | 0.002282 | 0.000698 | -1.18418 |
| 26 | 0.000064 | 0.005204 | -0.005510 | -0.001740 | 0.001758 | 0.000651 | -1.21892 |
| 28 | -0.000320 | 0.004550 | -0.004170 | -0.001640 | 0.001302 | 0.000611 | -1.25109 |
| 30 | -0.000630 | 0.003985 | -0.003050 | -0.001520 | 0.0009 | 0.000577 | -1.28104 |
| 32 | -0.000870 | 0.003496 | -0.002100 | -0.001390 | 0.000545 | 0.000544 | -1.30905 |
| 34 | -0.001020 | 0.003193 | -0.001320 | -0.001410 | 0.000223 | 0.000551 | -1.33537 |
| 36 | -0.001280 | 0.002579 | -0.000640 | -0.000970 | -0.00026 | 0.000445 | -1.36018 |
| 38 | -0.001400 | 0.002248 | -7.3E-05 | -0.000830 | -0.00026 | 0.000409 | -1.38364 |
| 40 | -0.001510 | 0.001912 | 0.0003340 | -0.000690 | -0.0004 | 0.000389 | -1.40591 |
The \( i_{k,a}^{(0)} \) value can be expressed via this relation:

\[
i_{k,a}^{(0)} \approx 10^E
\]  

(15)

For a strip with the coordinate \( x \) deformation computation the following relation is valid:

\[
w_k(x) = H \sum_{s=1}^{S} p_s^{(0)} \cdot I_{k,s}(x)
\]

(16)

where integral:

\[
I_{k,s}(x_k) = \int_{y_{sa}}^{y_{sb}} \int_{x_{sa}}^{x_{sb}} \sqrt{1 - \left( \frac{x}{x_{k,a}} \right)^2} \cdot \frac{dy \cdot dx}{\left( (x_k - x)^2 + (y - y_k)^2 \right)^{3/2}}
\]

(17)

This integral can be modified into the one-dimensional integral in form:

\[
I_{k,s}(x_k) = -\frac{1}{|x_{k,a}|} \int_{y_{sa}}^{y_{sb}} \sqrt{\frac{1}{(x_k - x)^2 + (y - y_k)^2}} \cdot Ln \left[ \frac{\sqrt{(x - x_k)^2 + (y - y_k)^2} - y_k + y_{sa}}{\sqrt{(x - x_k)^2 + (y - y_k)^2} - y_k + y_{sb}} \right] \cdot dx
\]

(18)

This integral has no analytical solution, so it is unavoidable to compute this integral numerically. In relation to other strips, the mentioned solution for strip by strip computation is time consuming.

We establish:

- the k-th strip length in the \( u, v \) system has the value of 1
- the k-th strip width in the \( u, v \) system has the value:

\[
b_k = \frac{y_{sa}}{x_{k,a}}
\]

(19)

and

\[
u_k = \frac{x_{k,a}}{x_{k,a}}
\]

(20)

where:

- \( x_k \) \( x \)-coordinate, in the \( x, y \) coordinate system on \( k \)th strip,
- \( x_{k,a} \) length of the \( k \)th strip,
- \( u_k \) \( u \)-coordinate, in the \( u, v \) coordinate system on \( k \)th strip.
Modified strip method specification for wheel/rail contact stress evaluation

\[ i_{k,s}(u_k) = \frac{q_{k,s}(u_k) + q_{k,s}(-u_k)}{q_{k,s}^{(0)}} \]  \hspace{1cm} (22)

\[ q_{k,s}^{(0)} = 2 \cdot \int_{-1}^{1} \int \frac{1-u^2}{\sqrt{u^2 + \left[2 \cdot (s-k-v) \cdot b_k \right]^2}} \cdot dv \cdot du \]  \hspace{1cm} (23)

\[ q_{k,s}(\chi) = \int_{0}^{1} \int \frac{1-u^2}{\sqrt{(\chi-u)^2 + \left[2 \cdot (s-k-v) \cdot b_k \right]^2}} \cdot dv \cdot du \]  \hspace{1cm} (24)

The \( q_{k,s}^{(0)} \) integral values and \( q_{k,s}(\chi) \) integral are utterly analytical.

Further, we compare the computational precision and computation speed gained by the Kalker’s method with the results gathered by other methods.

2.2. COMPUTATION RESULTS OBTAINED BY DIFFERENT METHODS

In the following graphs are maximum stresses in contact courses against lateral shift of the wheel along a rail head movement from a wheel rim (Fig. 11), maximum stresses in contact courses against lateral shift of the wheel along a rail head movement to a wheel rim (Fig. 12) as well as the contact patch values courses against lateral shift of the wheel along a rail head movement from a wheel rim (Fig. 13) and contact patch area courses against lateral shift of the wheel along a rail head movement to a wheel rim (Fig. 14).

Fig. 11. Maximum stresses \( (p_{\text{max}}) \) in contact courses against lateral shift of the wheel tread along a rail head move movement from a wheel rim

Fig. 12. Maximum stresses \( (p_{\text{max}}) \) in contact courses against lateral shift of the wheel tread along a rail head movement to a wheel rim
The computational time spent on the P5 computer, with 2GB RAMM, 3GHz frequency. Utilized methods: Strip method - level of tenth of second
- Kalker method with input of strip method results – level of second
- Kalker variation method - level of more than ten seconds.

2.3. CONCLUSIONS FROM NORMAL STRESS EVALUATION

The Strip Method is often used for rail/wheel contact area and contact normal stress evaluation. It presupposes quasi-static rolling. The principal idea of the theory is to take into consideration slim contact areas in the y-direction. The paper deals with the contact patch and normal stress computational time saving procedure when the computation accuracy is guaranteed.

The introduced method enables contact patches and contact stresses between railway wheel and rail [8] under decreased computational time consumption. Rules and procedures characteristic of the Strip method are preserved. The stress computation acceleration is in this case based on the algorithm for numerical solution of integrals. We included the local coordinate system with a presupposed semi-circular course of the normal stress for the purpose of the integral computation. This integral computation is needed for deformation in the middle of individual strips evaluation. The integral is solvable analytically. The input values for the separate spline parts and separate strips computation are possible to insert after analytical solution of the integral. The procedure application may bring the practical benefit for researchers and computation – analysis experts who are interested in the field of vehicle dynamics simulations, rail/wheel contact analysis, as well as new profiles on the base of geometric characteristics design [1,2].
3. TANGENTIAL STRESS

The part deals with the way of calculation of tangential stresses over non-elliptical contact patch, where is possible to utilize with advantage the Kalker’s simplified method FASTSIM. This method named FASTSTRIP is adapted for non-elliptical contact area calculated by means of the Strip method. The difference against the FASTSIM method is that the computation is executed along the strip separately, never mind whether the size of a strip is smaller or longer than the virtual ellipse border. This method is almost quick as FASTSIM and the results are similar to the CONTACT results. This method may be useful for rail vehicles in track dynamics computation.

3.1. TANGENTIAL STRESS EVALUATION

The rail/wheel contact relations for purposes of rail vehicles dynamics are often calculated by means of Hertz method [3] and Kalker simplified method applied in the program code FASTSIM [4]. Kalker’s variation method [5] used to be considered as an etalon for contact patch and contact stress between railway wheel and rail calculation. Normal stresses and contact patches areas are assessed with the program code CONTACT-NORM [5], tangential stresses and tangential forces with the program code CONTACT-TANG [5]. The computation with the Kalker’s variation method takes for longer time than the computation with the simplified method. This is the reason, that the variation method is not common widely used for rail vehicles dynamics computation and the simplified method is preferred for this purpose. The results gained with the simplified method are partially different (but acceptable) from the results gained with the variation method results. The most significant difference consists in the contact area shape and size calculated in program FASTSIM that presupposes always to be elliptical. The Strip method procedures [6, 7, 10, 11] give more opportunities to solve the contact with respect to non-elliptical contact patch. Our aim is to create the calculation procedure of „FASTSIM“ sort - we can name it „FASTSTRIP“ for calculation of stresses over non-elliptical contact area. (NFASTRSTRIP for normal stresses evaluation and TFASTSTRIP for tangential stresses evaluation). We derived the procedures for fast non-elliptical contact patch calculation [8] as a presupposition of tangential forces computation [9]. The results values of our brand new, in this article presented procedure are closer to the Kalker’s variation method [5] calculation results while the compute speed is similar to the compute speed of FASTSIM [4].

3.2. PREREQUISITES

At the beginning of tangential stresses evaluation, we calculate the moduli of shear elasticity for wheel and rail materials [3]:

...
where:

\[ G_1 = \frac{E_1}{2(1+\nu_1)} , \quad G_2 = \frac{E_2}{2(1+\nu_2)} , \quad G_{12} = \frac{1}{G_1 + \frac{1}{G_2}} \]  

(25)

\[ G_1, G_2 - \text{moduli of shear elasticity,} \]
\[ E_1, E_2 - \text{moduli of elasticity,} \]
\[ \nu_1, \nu_2 - \text{Poisson’s ratios,} \]

The contact area assembled from strips and normal stress above the strips are calculated with Strip method.

The output parameters from the Strip method that come into the modified procedure are:

- \( N \) - number of strips,
- \( y_i \) - centre of \( i \)-th strip coordinate,
- \( y_d \) - half-length of the strips,
- \( x_d \) - half-length of the \( i \)-th strip,
- \( \sigma_n \) - normal stress in the middle of \( i \)-th strip,
- \( A_{N,SS} \) - area of all strips.

We used the modified „FASTSTIM“ method for the tangential stresses computation. Fig. 15 shows the program dialog with graphical output of results.

**Fig. 15.** Plot of AreaNORM and AreaFASTSTRIP against wheelset treads profiles lateral movement

We use this method for the stresses in the non-elliptic contact patch area computation. Before the computation it is needed to compute or find out the virtual ellipse parameters:

- Slips \( s_x, s_y \) and spin \( \psi \) are calculated from the geometrical relations.
- For constants \( C_{11}, C_{22}, C_{23} \) determination, the imaginary elliptical contact patch with the \( b \) semi-axis:

\[ b = \frac{y_N - y_i}{2} + y_d \]  

(26)
the $a$ semi-axis:

$$ a = \frac{A_{\omega \xi}}{\pi \cdot b} \quad (27) $$

and with the ellipse centre coordinate $y_0$:

$$ y_0 = \frac{y_{1i} + y_{3i}}{2} \quad (28) $$

will be used.

For the semi axes proportion is valid:

$$ D = \frac{b}{a} \quad (29) $$

We will set $c_{1i}, c_{2i}, c_{3i}$ constants for given $d$ parameter and $\mu$ friction coefficient.

For $c_1, c_2, c_3$ constants are valid the relations:

$$ c_1 = \frac{9}{32}c_{1i}, \quad c_2 = \frac{9}{32}c_{2i}, \quad c_3 = \frac{3\sqrt{d}}{\pi}c_{3i} \quad (30) $$

We determine the number of splitting up the strips in the longitudinal direction:

$$ n_i = \text{Int}(8 \cdot a) \quad (31) $$

The tangential forces $T_x, T_y$ and a spin moment $M_z$ are set to be zero at the beginning.

### 4. MATHEMATICAL MODEL

The mathematical model is schematically depicted in the flow chart in Figure 16.

#### 4.1. CALCULATION AT THE I-TH STRIP (A)

For the i-th strip position coordinate in the imaginary ellipse area is valid:

$$ y_{ei} = \frac{y_i}{b} \quad (32) $$

and for its width is valid:
For tangential maximum stress is valid:

\[ T_0 = \mu \cdot p_{0i} \]  

(34)

where:

\( \mu \) is the friction coefficient.

We will calculate the following constants:

\[ C = \frac{G_{ij}}{T_0}, \]

(35)

\[ U_x = C \cdot C_1 \cdot s_x, \quad U_y = C \cdot C_2 \cdot s_y, \]

\[ F_x = C \cdot C_3 \cdot b \cdot \psi, \quad F_y = C \cdot C_4 \cdot a \cdot \psi \]

For the half-length of the strip is valid:

\[ a_i = \frac{x_i}{a} \]  

(36)

For a calculating step is valid:

\[ \delta_i = \frac{a_i}{n_x} \]  

(37)

and for area element is valid:

\[ A_x = y_{ol} \cdot \delta_x \]  

(38)

For the strip slip in the \( x \)-axis direction is valid:

\[ s_{xi} = U_x - F_x \cdot y_{ol} \]  

(39)
4.2. CALCULATION OVER THE LENGTH OF I-TH STRIP (B)

The \( p_x, p_y \) tangential stresses are set to be zero at the calculation beginning of the over the length of a strip.

The \( x_e \) coordinate is being changed in interval \( \left\{ a_{io} - \frac{\delta_e}{2} , a_{io} + \frac{\delta_e}{2} \right\} \) with a step of \( \delta_e \).

For current slip in the \( y \) direction is valid:

\[
\begin{align*}
    s_{yi} &= U_y + F_y \frac{p_x + x_i}{2} \\
    p_x &= p_x - s_{yi} \cdot (a_{io} - x_i) \\
    p_y &= p_y - s_{yi} \cdot (a_{io} - x_i)
\end{align*}
\]  

(40)

For tangential stresses over the area element with coordinates \((x_{ei}, y_{ei})\) is valid:

\[
\begin{align*}
    p_x &= p_x - s_{ei} \cdot (a_{io} - x_e) \\
    p_y &= p_y - s_{ei} \cdot (a_{io} - x_e)
\end{align*}
\]  

(41)

Then the stress amplitude and maximum feasible amplitude ratio will be calculated

\[
p = \sqrt{p_x^2 + p_y^2} \frac{a_{io}^2 - x_{ei}^2}{a_{io}^2 - x_{ei}^2}
\]  

(42)

If \( p > 1 \) then

\[
\begin{align*}
    p_x &= \frac{p_x}{p} \\
    p_y &= \frac{p_y}{p}
\end{align*}
\]  

(43)

We will calculate the \( T_{ex}, T_{ey} \) tangential forces and \( M_{ez} \) spin moment for an area element.

\[
\begin{align*}
    T_{ex} &= p_x \cdot A_x \\
    T_{ey} &= p_y \cdot A_y \\
    M_{ez} &= (p_x \cdot y_{io} - p_y \cdot x_e) \cdot A_e
\end{align*}
\]  

(44)

These forces and spin moment are added to \( T_x, T_y \) forces and \( M_z \) spin moment:

\[
\begin{align*}
    T_x &= T_{ex} + T_{e_x} \\
    T_y &= T_{ey} + T_{e_y} \\
    M_z &= M_{ez} + M_{ez}
\end{align*}
\]  

(45)

After calculation over the all strips the \( T_x, T_y \) forces and the \( M_z \) spin moment will be divided by the \( T_z = \frac{\pi}{2} \) constant.

\[
\begin{align*}
    T_x &= T_x \frac{T_z}{T_z} \\
    T_y &= T_y \frac{T_z}{T_z} \\
    M_z &= M_z \frac{M_z}{T_z}
\end{align*}
\]  

(46)

For true values of \( T_x, T_y \) and \( M_z \) spin moment obtaining, the values calculated from (22) are necessary to multiply by the constant \( T \) :

\[
T = N \cdot \mu
\]  

(47)
\[ T_x = T_x \cdot N \cdot \mu, \quad T_y = T_y \cdot N \cdot \mu, \quad M_z = M_z \cdot N \cdot \mu \]  

(48)

where: \( \mu \) is a friction coefficient, \( N \) is a normal force.

## 5. RESULTS AND VALIDATION

We analyzed the contact patch area, contact stress between the wheel equipped by S1002 tread profile and UIC60 rail head profile inclined by 1:40. The lateral shift is in interval of (cca -5mm to 5mm).

### 5.1. INPUT PARAMETERS

The wheel force is \( Q = 100.000N \). We used our fast strip method [8] for the computation of contact patches and contact stresses and we compared our results with the results obtained by Kalker’s Contact-NORM method [5].

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( F_n ) [N]</th>
<th>tan(( \Gamma ))</th>
<th>( \Gamma ) [rad]</th>
<th>( S_x ) [-]</th>
<th>( S_y ) [-]</th>
<th>( \Phi ) [rad/mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>100004</td>
<td>-0.008948</td>
<td>-0.00894776</td>
<td>0.001645</td>
<td>0</td>
<td>0.000019</td>
</tr>
<tr>
<td>-4</td>
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<td>-0.009655</td>
<td>-0.0096547</td>
<td>0.001295</td>
<td>0</td>
<td>0.000021</td>
</tr>
<tr>
<td>-3</td>
<td>100006</td>
<td>-0.01096</td>
<td>-0.01095956</td>
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In Tab. 2 are summarized input parameters: yw [mm] is lateral shift of wheels profiles over the rail heads profiles, Fn [N] is a normal force tan(\(\Gamma\)) is value of Tangent Gamma function that is the contact area angle tangent in the contact point. \(\Gamma\) [rad] is the same angle expressed in radians. \(S_x\) and \(S_y\) are the slips (creepages in x and y directions) and Phi is expressed in [rad/mm].

![Figure 17. Plot of Shift CONTACT-NORM, Shift NFASTSTRIP and Shift HERTZ against wheelset treads profiles lateral movement](image1)

![Figure 18. Plot of Area CONTACT-NORM, Area NFASTSTRIP and Area HERTZ against wheelset treads profiles lateral movement](image2)

![Figure 19. Plot of Pmax CONTACT-NORM, Pmax NFASTSTRIP and Pmax HERTZ against wheelset treads profiles lateral movement](image3)

![Figure 20. Plot of Pmax NFASTRSTRIP/NORM and Pmax HERTZ /NORM [-] proportional comparison of evaluated quantities](image4)

The difference against the FASTSIM method is that the computation is executed along the strip separately, never mind whether the size of a strip is smaller or longer than the virtual ellipse border.
5.2. RESULTS OF TANGENTIAL STRESSES CALCULATION

Tangential stresses, $T_x$, $T_y$ forces and $M_z$ moment for separate strips are computed by means of the „TFASTSTRIP“ method.

$T_x$, $T_y$, and $M_z$ computed by „CONTACT-TANG“, „FASTSIM“ and „TFASTSTRIP“ methods for a comparison.
5. CONCLUSIONS

The obtained results are effective from the point of view of computer time consumption. Our aim is to create the calculation procedure of “FASTSIM“ sort. We named this procedure for calculation of stresses over non-elliptical contact area “FASTSTRIP“. The results values are closer to the Kalker’s variation method results and the compute speed is similar to the compute speed of FASTSIM. This method is adapted for non-elliptical contact area calculated by means of the Strip method [8, 9]. This method utilizes the FASTSIM theory [4] as a calculation engine for tangential stress assessment. The calculation procedure is outlined in Figure 16. Here is drawn the flowchart with two program loops. These loops are in detail described in the part “Mathematical model”. Results and validation follows. In Table 1 are some input parameters, Figures 17, 18, 19 and 20 show the comparison of results gained by means of CONTACT-NORM [5] and our calculation procedure [8] shift, area and p_{max}. They express the reality, that the ground input parameters for tangential forces calculations are mutually very close.

Figures 21, 22 and 22 give results of tangential stresses calculation for input parameters. The curves of dependencies \((T_x, T_y, M_z)\) calculated with CONTACT-TANG [4] and FASTSTRIP [9] are shown in graphs. For better resolution are these curves shown in Figures 20, 24, 25 and 26 as comparative proportional curves. The meaning or importance of the procedure FASTSTRIP (NFASTSTRIP and TFASTSTRIP) for us or somebody who writes his/her own code is in the fact, that this procedure can be implemented into the code for rail vehicles dynamics computation with the advantage of fast computations.

ACKNOWLEDGEMENT

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Research-Educational Center of Rail Vehicles (VVCKV)

References


MODYFIKACJA METODY PASKOWEJ DO ANALIZY NAPRĘŻEŃ W KONTAKCIE KOŁA Z SZYNĄ

Streszczenie: Metoda paskowa (Strip Method) jest powszechnie używana w analizie naprężeń kontaktowych w układzie koło/szyna. W artykule przedstawiono zmodyfikowaną procedurę obliczeń z zachowaniem dokładności uzyskiwanych wyników. Modyfikacja pozwoliła na znaczne zmniejszenie czasu obliczeń. Oblczenia kontaktowych naprężeń normalnych wykonywane są w lokalnym układzie współrzędnych co umożliwia obliczenia naprężeń w pojedynczych paskach kontaktowej powierzchni. Całkowanie wykonywane jest metodą analityczną. Opracowany program TFASTSTRIP bazuje na programie Kalkers FASTSIM. Parametry (funkcje) wejściowe do obliczeń w elementarnym pasku są aproksymowane spalinami, co pozwala na otrzymanie analitycznego rozwiązania.

Słowa kluczowe: kontakt koła z szyną, kontaktowe naprężenia, modyfikacja metody paskowej, optymalizacja czasu obliczeń, Contact-TANG, Contact-NORM, FASTSIM